**MEMO**

**TO:** Dr. Crane

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**DATE:** March 21, 2022

**SUBJECT:** ME 472 – Spring back analysis

## Introduction

In this report, we are looking to predict the spring back of a 19 mm long, 3.175 mm diameter bar after it has been bent around mandrels of various sizes. This will be done through comparing hand calculations, finite element analysis, and experimental results. ANSYS Workbench, SpaceClaim, ANSYS Mechanical, and MATLAB R2022a were used to complete this analysis.

## Methods

Initially for the geometry, a 19-millimeter line was created and then was turned into a 3.175-millimeter diameter beam using a circular beam profile. In Figure 1 below we can see where the fixed support and radial displacement were placed on the beam. A cylindrical coordinate system, was also created to aid in

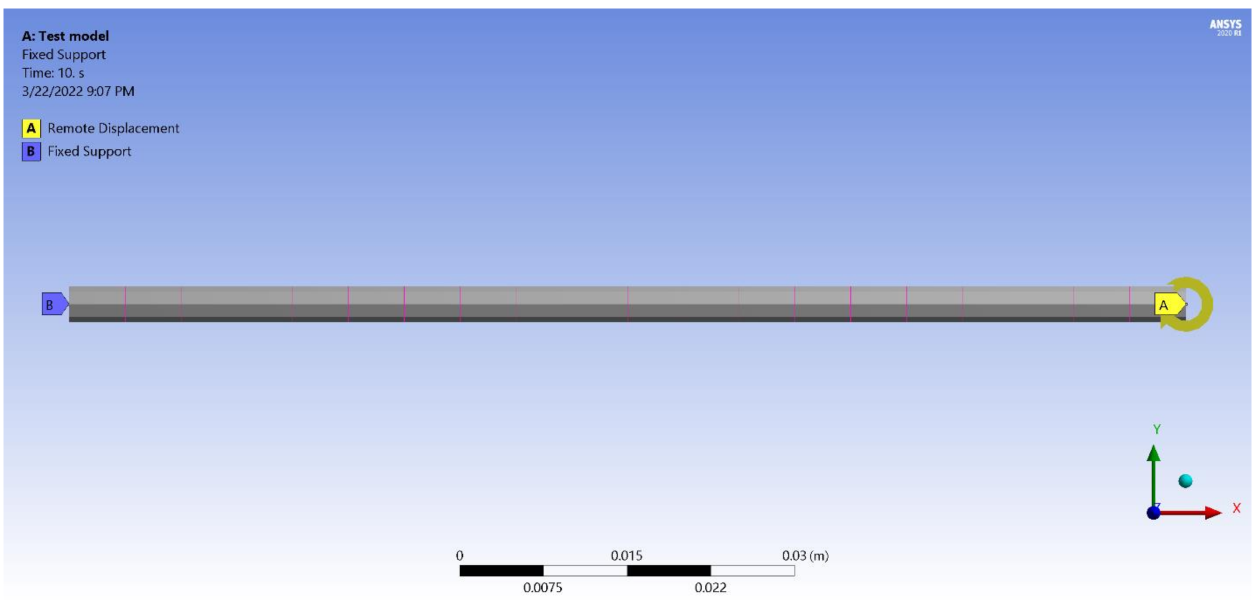


*Figure 1: Image showing the location of the fixed support and the remote displacement. The rotation of the arrow shows the direction of the rotation for the remote displacement.*

determining the radius of the rod after it had sprung back. The origin of the cylindrical coordinate system is at the same point that the fixed support is, which is seen in Figure 1 above.

After large deflections were turned on under the Analysis Settings portion of ANSYS Mechanical, we began to apply the remote displacements. Table 1 below shows the $\theta_{max}$ that was applied to simulate wrapping the rod around the corresponding mandrel diameter. The equation to solve for $\theta_{max}$ is as follows:

$$-\theta_{max} = \frac{2l}{D_{man} + d_{wire}} \tag{1}$$

*Table 1: Table showing the theta max required for specific mandrel sizes when the diameter of the rod is 3.175 mm.*

| Mandrel Size | Corresponding $\theta_{max}$ |
|---|---|
| 8.5598 mm | 341.7787° |
| 14.1351 mm | 231.6974° |
| 22.0345 mm | 159.095° |
| 31.369 mm | 116.1042° |
| 50.5206 mm | 74.6934° |
| 97.2058 mm | 39.9549° |

After the maximum displacement had been placed, since the Y axis Direction Deformation Solution output the radial deformation of the rod after it had sprung back in meters, multiplying it by two allowed us to obtain the diameter of the rod after it had sprung back. Design parameters were then used to

## Material Properties

Per the instructions, Aluminum was selected as the material for the rod and the values for the material properties were input based on values that were provided to us by Dr. Crane. Those values are shown below in Figure 2.

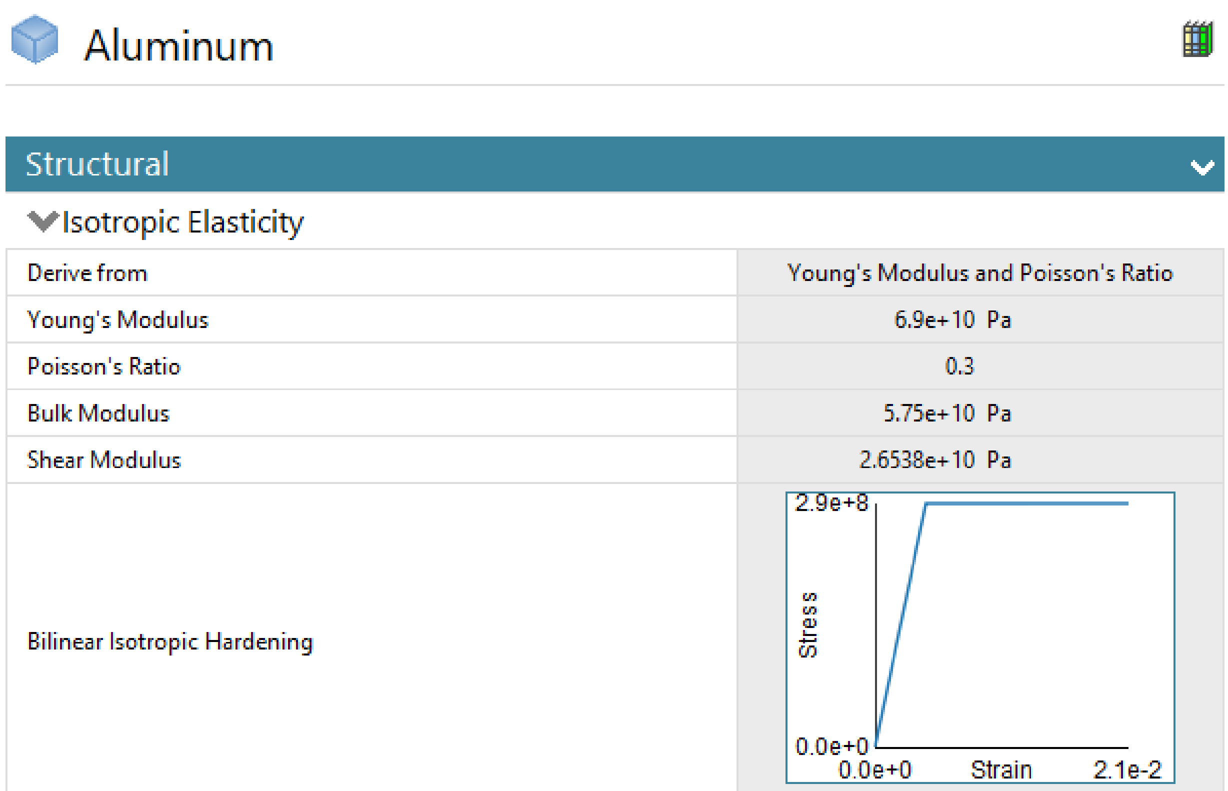


*Figure 2: Material properties for Aluminum as used in this analysis.*

## Validation/Verification of the Model

In an attempt to help with validation and verification of the model, hand calculations were performed. They were derived in a document on the FEA Lecture Materials section under the Content tab of the ME EN 472 Learning Suite page. The derivation will not be shown here, but the equations themselves will be shown. Table 2 following the equations is an explanation for the various variables.

$$\epsilon_r = \epsilon_{max} + \epsilon_e = -\frac{d_{wire}}{D_{man} + d_{wire}} + \epsilon_e \tag{2}$$

$$M_0 = \frac{4r^3 S_y}{3} \tag{3}$$

$$\epsilon_e = \frac{M_0 r}{IE} = \frac{16 S_y}{3\pi E} \tag{4}$$

$$\theta_r = \frac{\epsilon_r l}{r_{wire}} = \frac{2\epsilon_r l}{d_{wire}} \tag{5}$$

$$D_r = \frac{2l}{\theta_r} \tag{6}$$

$$D_{IDresidual} = D_r - d_{wire} \tag{7}$$

*Table 2: Description of variables and listing of the value used for said variables in the hand calculation.*

| Variable | What it is | Value used |
|---|---|---|
| $d_{wire}$ | Diameter of the wire (m) | 0.003175 m |
| $D_{man}$ | Diameter of the mandrel (m) | 0.0972058 m |
| $S_y$ | Yield Strength (Pa) | $6.9 \cdot 10^8\ Pa$ |
| $E$ | Young's Modulus (Pa) | $6.9 \cdot 10^{10}\ Pa$ |
| $D_{IDresidual}$ | (Diameter after rod has sprung back) | 0.1328 m |
| $l$ | Length | 0.019 m |

As is shown in Table 2 above, from the hand calculations we got that the diameter of the rod after it had sprung back should be 0.1329 m. Using that value, we proceeded to do a mesh convergence, and Table 3 below shows the results of that study. Based on the results from the study, a mesh element size of 0.01 m was selected as we proceeded through our study.

*Table 3: Results from mesh convergence study performed.*

| Mesh Element Size | Diameter after rod has sprung back |
|---|---|
| 0.01 m | 129.22 mm |
| 0.005 m | 129.19 mm |

Also, as shown in Table 3, while the diameter of the rod after spring back did converge, it did not converge on the value that we obtained from our hand calculations. Based on the numbers the value we obtained from ANSYS Mechanical was 2.71% less than the hand calculations, but with a value that small, we will assume the difference to be negligible and a mesh element size of 0.005 m will be used.

## Results/Conclusions

Below is Table 4 comparing the mandrel's diameter to the spring back values of the rod obtained through experiments, hand-calculations, and the FEA. As can be seen, values from the FEA analysis were not obtained for each of the mandrel sizes as the model was not able to converge on a final value.

*Table 4: Table summarizing the results of the various spring back diameters found through the different methods.*

| Mandrel Size | Hand Calculated spring back dia. | % change from hand calculated | Experimental spring back dia. | % change from Experimental | FEA spring back dia. | % change from FEA |
|---|---|---|---|---|---|---|
| 8.5598 mm | 0.0152 m | 77.90% | 0.0085598 m | 8.90% | - | - |
| 14.1351 mm | 0.0212 m | 49.88% | 0.015113 m | 6.92% | - | - |
| 22.0345 mm | 0.0299 m | 35.69% | 0.240284 m | 9.05% | - | - |
| 31.369 mm | 0.0406 m | 29.51% | 0.035179 m | 12.15% | - | - |
| 50.5206 mm | 0.0642 m | 27.15 % | 0.0623316 m | 23.38% | 0.06104 | 20.82% |
| 97.2058 mm | 0.1329 m | 36.63% | 0.134112 m | 37.97% | 0.12919 m | 32.90% |

Figure 3 below shows a plot of the mandrel size vs spring back diameter obtained through various different methods. As can be seen, the points are all very close suggesting that the experimental results are quite accurate. Also, with the FEA analysis values being so close to the hand calculated and experimental values for those values that were obtained, it can be adequately assumed that there would be similarly close values for the remaining four mandrel sizes as well.

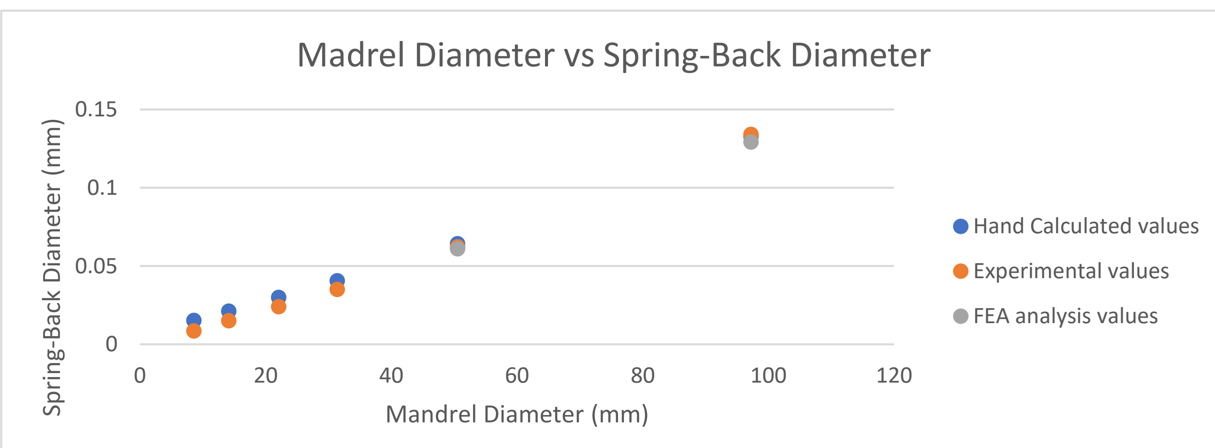


*Figure 3: Graph showing mandrel diameter vs spring-back diameter for the diameters that were obtained.*